Table 3.1
Comparison of third-order elastic constants determined from Hugoniot elastic limit measurements to other measurements*

Material	S_{xx} %	C_{xxx}^{HEL} TPa	C_{xxx}** TPa	C_{xxx}^{HEL}/C_{xxx}
[111] NaCl	1.8	−0.2 [70M2]	−0.175 [69B1]	1.14
[100] MgO	2.5	−6.0 [66A2]	−4.9 [69B1]	1.22
[100] MgO	0.64–0.83	−6.2 [77G2]	−4.9 [69B1]	1.27
[100] InSb***	2.9	−0.58 [65K2]	−0.31 [69B1]	1.87
[110] InSb***	2.4	−0.53 [65K2]	−0.59 [69B1]	0.90
[100] Silicon	4.6–5.7	−1.0 [71G6]	−0.83 [69B1]	1.20
[110] Silicon	2.1–2.9	−1.8 [71G6]	−1.50 [69B1]	1.20
[111] Silicon	2.5	−1.6 [71G6]	−1.33 [69B1]	1.20
X-cut quartz	5.6–6.8	−0.56 [67F1]	−0.30 [72G3]	1.87
X-cut quartz	5.3–8.1	−0.54 [62W1]	−0.30 [72G3]	1.80
Z-cut quartz	6.7–9.7	−1.4 [67F1]	−0.82 [69B1]	1.71
Z-cut quartz	4.5	−1.2 [62W1]	−0.82 [69B1]	1.46
[111] Ge	1.35, 2.2	−1.6 [66G1]	−1.12 [69B1]	1.43
[111] Ge	2.6	−1.3 [72G5]	−1.12 [69B1]	1.2
[110] Ge	3.2	−1.6 [72G5]	−1.31 [69B1]	1.22
[110] Ge	4.5	−0.81 [72G5]	0.73 [69B1]	1.11
a axis CdS***	3.6	−0.30 [66K1]	—	—
c axis CdS***	4.3	−0.40 [66K1]	—	—
[100] TiO_2	2.3	−4.0 [68A2]	—	—
[001] TiO_2	2.1	−2.0 [68A2]	—	—
X-cut $CaCO_3$	1.5	−1.0 [68A2]	0.579 ± 0.017 [69B1]	1.75
Z-cut $CaCO_3$	2.2	−0.4 [68A2]	0.498 ± 0.13 [69B1]	0.8

* The x direction is taken to be the wave propagation direction.
** These values are as determined from the various ultrasonic investigations.
*** These shock data relate to the limiting elastic compression just prior to a polymorphic phase transformation.

ultrasonically-determined constant. This indicates that the fourth-order terms contribute significantly to the stress-strain response of these materials, even at strains of only a few per cent.

Relatively few measurements of higher-order elastic response have been made by the method of shock compression. Those that are available are summarized and compared to ultrasonic measurements in table 3.2. The agreement of third-order constants determined by the two methods is, in general, quite good. The third-order constants determined for vitreous silica by shock compression are probably the most accurate available for that material, with those obtained at high temperature illustrating a unique capability of the method. The shock experiments are the only source of fourth-order constants for most of the materials.

The work reviewed in this section shows that shock-compression experiments provide an effective method for determination of nonlinear elastic properties and that, by the same token, the effects of nonlinear elastic response should generally be taken into account in investigations of shock compression (see, e.g., Asay et al. [72A2]). Fourth-order contributions are readily apparent, but few coefficients have been measured. Tabulations of third-order constants are given in the Landölt–Börnstein tables [69B1] and by Zarembo and Krasil'nikov [71Z1].

Linear elasticity theory. It is convenient at this point to note, for future reference, that the linear approximation to eq. (3.1) takes the form $t_{ij} = C_{ijkl}S_{kl}$. When the material exhibits isotropic

Table 3.2
Longitudinal elastic constants determined ultrasonically and under shock compression*

Material/Method	C_{xx}, GPa	C_{xxx}, GPa	C_{xxxx}, GPa
Vitreous silica-GE151			
shock [72G2]	77.4	+550	+11000
sonic	77.7 [74F1]	+603 [73Y2]**	—
Vitreous silica-Dynasil 1000			
shock [77A3]	77.5	+550	+12000
shock (473K) [77A3]	81.2 ± 0.4	+660	+14000
X-cut quartz			
sonic	86.8 [69B1]	−210 [66T2]	—
shock [72G3]	86.8 ± 1	−300 ± 30	+7500 ± 2500
X-cut Al_2O_3			
sonic	49.4 [69B1]	−3900 [70H1, 68G1]	—
shock [72G2]	—	−3300 ± 300	50000 ± 15000
Z-cut Al_2O_3			
sonic	49.6 [69B1]	3300 [70H1]	—
		3100 [68G1]	—
shock [72G2]	—	3250 ± 100	—
shock [72G2]	—	3300 ± 300	50000 ± 15000

* Measurements are made at room temperature, ~ 300 K, except for the single high-temperature measurement on vitreous silica. The wave propagation direction is taken to be the x direction.

** Cantrell and Breazeale [78C1] have recently reported third-order elastic constants of other vitreous silica materials which range from 670 to 730 GPa.

symmetry, as is usually assumed to be the case for polycrystalline substances, this relation yields the results

$$t_1 = (K + \tfrac{4}{3}\mu)S_1, \qquad t_2 = t_3 = (K - \tfrac{2}{3}\mu)S_1, \qquad t_{ij} = 0 \text{ for } i \neq j \tag{3.4}$$

(in Voigt notation) for states of uniaxial strain. In these relations the constants K and μ are called the bulk and shear moduli, respectively. Application of equations (2.8) and (2.9) shows that the pressure and maximum shear stress are $p = -KS_1$ and $\tau = \mu S_1$, respectively, so that the part $-KS_1$ of the applied stress is borne by the resistance of the material to compression (or expansion) while the part $\frac{4}{3}\mu S_1$ is borne by its resistance to shear.

Transverse waves. Experiments involving plane waves of uniaxial strain produce proportional strain in compression and shear. If applied to a linearly elastic and isotropic solid, they permit determination only of the combination $K + \frac{4}{3}\mu$ of the bulk and shear moduli. An additional measurement is required to separate the effects of resistance to compression and shear, and thus complete the characterization of even this simplest solid. The situation is more complicated when the material is nonlinear and/or anisotropic, but the basic problem is the same: the uniaxial strain experiment does not allow independent variation of compression and shear strain. Theoretical analyses of combined longitudinal and shear motions have been available for some time (see, e.g., Davison [66D1, 68D1]), but, until recently, shock-loading apparatus and instrumentation have been limited to production and measurement of longitudinal motion. Recent development of methods for producing and measuring transverse components of velocity and displacement histories promise significant enhancement of existing capabilities for studying shear stress in shock-loaded solids.